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CAPITAL ASSET PRICING MODEL TESTS IN A TERM STRUCTURE CONTEXT

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1.1 Introduction

This paper develops and presents new empirical tests of a set of models of equilibrium rates of return on capital assets which are prominent in the finance literature. The most elaborate model is the intertemporal capital asset pricing model (ICAPM) proposed by Merton [36] which extends the earlier capital asset pricing model (CAPM) of Sharpe [50] and Lintner [31]. That model represents the equilibrium expected rate of return on any asset as a weighted average of the return on a riskless bond and the return on a portfolio of all risky assets in their outstanding market value proportions. Both the underlying derivations and the logic of tests of the CAPM assume the probability distributions of rates of return on all risky assets are stationary and that the return on the riskless asset is constant, or at least nonstochastic. Merton's ICAPM allows for the additional risks that investors bear due to unforseen changes in the riskless rate or other unanticipated changes in the investor's opportunity set due to shifting "state variables".

Despite its elegance and intuitive appeal, the ICAPM has not so far been thoroughly tested mainly because of difficulties in identifying the additional risk factors or state variables and finding appropriate proxies to represent them. Recently, however, Breeden [8] has proposed a reinterpretation of Merton's ICAPM that should, in principle at least, lend structure to its empirical testing. Breeden notes that while Merton's analysis of the ICAPM focuses on the choice of an optimal portfolio by the investor, it simultaneously solves for the individual's optimal current consumption as a function of the state variables. Breeden then goes on to show that observed

consumption per capita may be taken as an "instrumental variable"[1] for unobserved and undefined state variables.

Formally, Breeden's model can be expressed as a single-index model:

$$E(R_j) = R_F + \beta_j[E(R_M) - R_F] \qquad (1)$$

$$\beta_j = \frac{\mathrm{cov}(R_j, \frac{\Delta C}{C})}{\mathrm{cov}(R_M, \frac{\Delta C}{C})}$$

where: $E(R_j)$ is the equilibrium expected rate of return on asset j, conditional on R_F, the (instantaneous) riskless rate of interest, or more generally, conditional on the entire set of state variables;[2] $E(R_M)$ is the expected rate of return on a market index. It is not required that M be "the" market; $C(t)$ is optimal per capita consumption, where "optimal" means satisfying the envelope condition of the representative consumer's dynamic optimization problem (Merton [35]) – $C(t)$ in this sense is essentially "permanent consumption."

Because the covariance operator is linear, a linear asset pricing model such as that in (1) arises if any instrument (say $\xi(t)$) is substituted for $C(t)$, so long as that instrument satisfies the condition:

$$\left.\frac{-U_{CC}}{U_C}\right|_{\xi} = \frac{E(R_M) - R_F}{\mathrm{cov}(R_M, \xi)} \equiv g(\xi) \quad ,$$

where $-U_{CC}/U_C$ is the Arrow-Pratt measure of absolute risk aversion. In words, any "optimizing" variable or "utility index" such as the weights that define the "true" efficient portfolio can replace "the" market or optimal consumption. In the tests presented here, the utility index is treated as unobservable.

The single-index "consumption beta" β_j in (1) can be interpreted in two different ways--as a constant or as a varying parameter. If the opportunity set is stable, wealth is an adequate descriptor of utility; and if the representative individual's utility function is also isoelastic, β_j will be constant. In that case, (1) differs from the traditional CAPM only by the substitution of the utility index or per capita consumption $C(t)$ for the market index as a proxy for wealth. On the other hand, if (nondeterministic) shifts in the opportunity set occur,[3] they will be impounded in the consumption beta, so that, except in special cases, it must be interpreted as varying stochastically. $E(R_j)$ in (1) must then be understood as conditional on the opportunity set shift variables. If utility is not isoelastic, aggregate wealth changes may also induce variations in the beta.

1.2 The Testing Strategy: Nesting the Alternative Hypotheses

The stochastic beta model under the second interpretation of (1) includes the stable beta as a special case; i.e., it "nests" the stable beta model. Additionally, it will be shown that the stable beta model (1) nests the traditional CAPM. The rationale for nesting models in this manner is that "it takes a model to beat a model," so that tests are best structured to focus on the incremental explanatory power of the increasingly more elaborate models. Indeed, outside this nested hypothesis framework, it is not clear what it even means to reject (say) the traditional CAPM against a general unspecified alternative.[4]

The sequencing of the nested tests is most easily represented in the flow chart of Figure 1. The tests begin with the observable means and

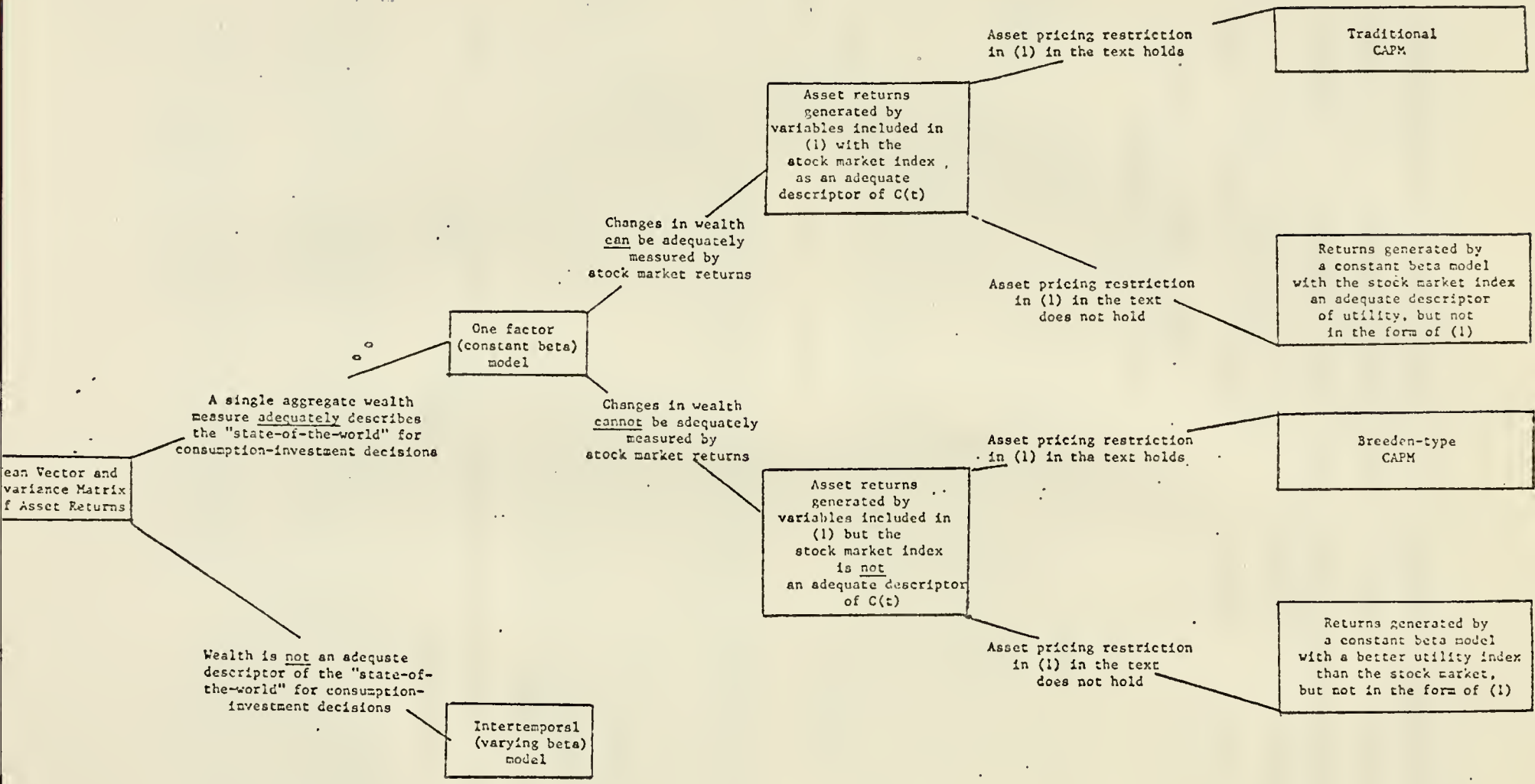
ean Vector and
variance Matrix
f Asset Returns
A single aggregate wealth measure adequately describes the "state-of-the-world" for consumption-investment decisions
Wealth is not an adequate descriptor of the "state-of-the-world" for consumption-investment decisions
One factor (constant beta) model
Intertemporal (varying beta) model
Changes in wealth can be adequately measured by stock market returns
Changes in wealth cannot be adequately measured by stock market returns
Asset returns generated by variables included in (1) with the stock market index, as an adequate descriptor of C(t)
Asset returns generated by variables included in (1) but the stock market index is not an adequate descriptor of C(t)
Asset pricing restriction in (1) in the text holds
Traditional CAPM
Asset pricing restriction in (1) in the text does not hold
Returns generated by a constant beta model with the stock market index an adequate descriptor of utility, but not in the form of (1)
Asset pricing restriction in (1) in tha text holds
Breeden-type CAPM
Asset pricing restriction in (1) in the text does not hold
Returns generated by a constant beta model with a better utility index than the stock market, but not in the form of (1)

Figure 1

covariances of asset returns upon which the asset pricing models impose a testable structure. Two initial branches are possible--in one, the state-of-the-world as viewed by the investor in making consumption-investment decisions can be described by a single aggregate wealth measure, i.e., the underlying state space is one-dimensional. In the other, there is multidimensional systematic uncertainty about opportunity set changes of concern to the investor, and such uncertainty cannot be spanned by one wealth index.

In the first branch, if opportunity set changes are not significant but consumption is superior to the observed market index as a measure of wealth, the stable "consumption" beta model (1) dominates the form of the traditional CAPM. On the other hand, if consumption is not a superior measure of wealth, the form of the traditional CAPM does as well as its more elaborate "consumption" beta counterpart in explaining asset returns. The thrust of the final hypothesis test shown in the first branch can be easily seen by looking at (1). In the single-index model, the variable R_F occurs twice in the equation--once with a coefficient of 1 and once with the same coefficient (β_j) as the expected return on the market index. Thus, if we think of a multivariate regression corresponding to (1) which involves R_j, R_F, and R_M, there is a within-equation restriction on the coefficients of R_F and R_M. This restriction is referred to as that implied on asset returns by the asset pricing model, and it is the subject of the last level of testing in the first branch.

In the second branch, opportunity set changes are taken into account, in addition to ("true") wealth, by investors in making their consumption-investment decisions. Since opportunity set changes are only of concern in

an intertemporal context, these changes will manifest themselves in a stochastic beta in the Breeden-form of the Merton ICAPM. It is also possible to ask whether the asset pricing restrictions on the coefficients of (1) obtain in the stochastic beta case, but since it is not clear what it means for a restriction to hold on stochastic coefficients, and since the results here show that the stable (real) beta hypothesis is not rejected in the term structure context, those restrictions are not stated here.

1.3 Bonds and the Term Structure

The asset pricing tests are performed here across a vector of default-free bond returns of differing maturities. As Cox, Ingersoll, and Ross [1977] have pointed out in a general equilibrium setting, default-free bonds are risky in real terms only because of stochastic changes in wealth and the opportunity set over their term to maturity. In contrast, much of the variation in common stock returns is attributable to firm-specific technology, so that, in the absence of some unexplained maturity-specific demand factors, bonds seem more appropriate securities on which to perform ICAPM tests in spite of (actually because of) the relatively high correlation of their returns across maturities (if there were literally no maturity-specific components of bond returns, the rank of the correlation matrix of bond returns would be bounded above by the dimensionality of the vector of underlying state variables). It turns out that, as the maturity of the bonds increases, their returns seem to behave "more like" common stock market indices, so that a cross-section of bond maturities covers the spectrum from one-month Treasury bills to stocks.

Default-free discount bonds differ only along a single dimension, that is, maturity. As a result, various term structure models imply different cross-maturity restrictions on the (possibly stochastic) β_j's . By contrast, cross-sectional restrictions beyond those possibly implied by the intertemporal CAPM would, in general, have little meaning in the case of common stocks. The interpretation of β_j's in a term structure context is of more than passing interest here--when the β's vary stochastically, the term structure context aids a good deal in modeling the state-dependent variation.

Basing the asset pricing tests on bond returns also forces us to properly account for inflation in asset pricing models like (1)--the "nominal versus real returns" issue is usually glossed over in asset pricing models applied to stock returns. Here, model (1) has been restated to include inflation uncertainty, and the tests take into account the inflation uncertainty in the model.

The results here seem to suggest that real opportunity set changes are negligible in the pricing of bonds, i.e., the "real" β_j in (1) are not stochastic over the time period examined: March 1959 to December 1978. Interestingly, when term premiums are stated in terms of the excess of holding period returns of longer-term bonds over short-term bonds, this means that we cannot detect variations in real term premiums over that time period. However, it does seem that stock market returns are not adequate descriptors of utility, i.e., a consumption-type model significantly improves upon the form of the traditional CAPM. Finally, it appears that the within-equation asset pricing restrictions cannot be rejected for all but the shortest maturity bonds, which is interesting because the time

series behavior of bond returns at longer maturities looks most like that of common stocks for which these types of asset pricing restrictions have been rejected in the past. Section 6 contains some speculation on the reason for the differences in results.

The methodology developed here can contribute a straightforward estimate of the relative importance of "nominal" and "real" factors in the determination of nominal bond premiums. The issue was addressed by Fama [12], but various econometric problems make his results difficult to interpret. Nevertheless, the results here generally support Fama's contention that nominal factors are predominant at the short end of the maturity spectrum, but not for the longer maturity classes.

2. Construction of the Test

2.1 Basic Approach

The consumption ICAPM (1) can be derived in its most general form with a stochastic β_j, in any of the economies considered by Merton [35], Cox, Ingersoll and Ross [10], Brock [9], Breeden [8] or Garman [16]. However, as pointed out in the Introduction, conditional on the stationarity of β_j, (1) will differ from the traditional CAPM only in its proposed wealth surrogate. Hence in the constant β_j case, the underlying economies must be appropriately restricted so that state variables other than wealth are not important. Indeed, as long as that restriction holds, it will be shown below that consumption or any alternative wealth surrogate to be used in the tests need not even be observed.

First, the test is derived where the consumption surrogate for wealth, C(t), can be observed. The following simple two-equation system forms the basis of the test methodology:

$$R_j(t) - \beta^j_{12} R_M(t) - \gamma^j_{11} = u^j_1(t) \qquad (2a)$$

$$R_M(t) - \gamma_{21} - \gamma_{22} \frac{\Delta C}{C}(t) = u_2(t) \quad , \qquad (2b)$$

where the disturbance $\underline{u}'(t) = [u^j_1(t), u_2(t)]$ is assumed to be independently and identically distributed (through time), with $E\underline{u} = \underline{0}$, $E\underline{u}\underline{u}' = \Sigma$ with Σ a positive definite symmetric matrix, and $E[\underline{u}|(\Delta C/C)(t)] = 0$. Provided $\gamma_{22} \neq 0$, the rank condition for identification of both equations will be satisfied.

It will be shown below that (2) bears the same relation to the equilibrium consumption model as the well-known market model bears to the traditional CAPM. Indeed, (2a) considered separately <u>is</u> the market model. (2) is a <u>statistical</u> representation of the relation between the variables $R_j(t)$, $R_M(t)$, and $(\Delta C/C)(t)$. It will be shown that the various financial models, including (1), which are nested in the diagram of Section 1, imply certain restrictions on (2).

As a statistical model, (2) obviously implies a good deal of structure in the data. By analogy with the market model, if $[R_j(t), R_M(t), (\Delta C/C)(t)]$ belong to a multivariate distribution possessing the linearity properties of the multivariate normal or multivariate Student-t, then $E[u_1|(\Delta C/C)(t)] = E[u_2|(\Delta C/C)(t)] = 0$. Given the structure of (2), this is sufficient for identification. The distributional properties of the data and the adequacy of (2b) will be examined below.

It will now be shown that β_{12}^j in (2a) corresponds to the β_j in the consumption model (1). The reduced form of (2) is:

$$R_j(t) - \Pi_1 - \Pi_2 \frac{\Delta C}{C}(t) = v_1^j(t) \tag{3a}$$

$$R_M(t) - \Pi_3 - \Pi_4 \frac{\Delta C}{C}(t) = v_2(t) \tag{3b}$$

where

$$\Pi_1 = \gamma_{11}^j + \beta_{12}^j \gamma_{21}$$

$$\Pi_2 = \beta_{12}^j \gamma_{22}$$

$$\Pi_3 = \gamma_{21}$$

$$\Pi_4 = \gamma_{22}$$

for the exactly identified system. Then:

$$\beta_{12}^j = \frac{\Pi_2}{\Pi_4} = \frac{\text{cov}\,(R_j, \frac{\Delta C}{C})}{\text{cov}\,(R_M, \frac{\Delta C}{C})} \quad . \tag{4}$$

It can be seen that the population parameter β_{12}^j in (4) is equivalent to the β_j of the model (1). Also:

$$\Pi_1 = \gamma_{11}^j + \beta_{12}^j \gamma_{21}$$

$$\Rightarrow \quad \gamma_{11}^j = \Pi_1 - \beta_{12}^j \gamma_{21} \tag{5}$$

$$= E(R_j) - \beta_{12}^j E(R_M) \qquad \text{(from (2a))} \quad .$$

The equilibrium model (1) can be written as:

$$R_F(1 - \beta_j) = E(R_j) - \beta_j E(R_M) \quad . \tag{6}$$

If the null hypothesis H_0 is that the equilibrium model (1) holds, then it follows from (5) and (6) and the equality of β_j and β_{12}^j that H_0 is:

$$H_0: \quad \gamma_{11}^j = R_F(1 - \beta_j) \quad . \tag{7}$$

In (2), the matrix B of coefficients of the endogenous variables $R_j(t)$ and $R_M(t)$ is upper triangular, i.e.,

$$B = \begin{bmatrix} 1 & \beta_{12}^j \\ 0 & 1 \end{bmatrix} \quad .$$

Hence, if the covariance matrix Σ of $\underline{u}$ were diagonal, the system would be fully recursive, and the traditional market model "beta" given by (2a) alone would equal that given by the two equation system (2a) and (2b). That is, (2b) would be simply a "change of variables" equation and the OLS estimator of β_{12}^j in (2a) would be unbiased. (Of course, the OLS estimation of beta and (say) the instrumental variable estimation (4) might differ in efficiency.)

Hence, a test of the difference between the consumption model beta and the traditional market model beta is, in econometric terms, a test of specification in system (2a) (conditional on the exclusion restrictions in (2)).

Equation (2) in effect posits $\Delta C/C$ as an instrument for R_M. When (2a) is considered alone as a statistical model generating the asset returns $\{R_j(t), R_M(t)\}$ from a bivariate normal distribution, the linearity of the bivariate normal implies:

$$E(R_j|R_M) = [E(R_j) - \beta^a_{12}E(R_M)] + \beta^a_{12}R_M$$

where: $$\beta^a_{12} = \frac{cov(R_j,R_M)}{var(R_M)}$$

When the linear specification (2b) is added to (2a), there are three variables present in the system: $R_j(t)$, $R_M(t)$ and $\frac{\Delta C}{C}(t)$. Yet interest lies in only two of these, $R_j(t)$ and $R_M(t)$, which are both determined within the system (2). To understand the relation between $R_j(t)$ and $R_M(t)$, given that they are now determined simultaneously with $\Delta C/C(t)$, substitute (2b) into (2a), and omit the subscript j for simplicity and temporarily let β^b_{12} be the β_{12} generated by (2):

$$R(t) = (\gamma_{11} + \beta^b_{12}\gamma_{21}) + \beta^b_{12}\gamma_{22}\frac{\Delta C}{C}(t) + v_1(t) \qquad (8a)$$

$$R_M(t) = \gamma_{21} + \gamma_{22}\frac{\Delta C}{C}(t) + v_2(t) \qquad (8b)$$

from which it follows that, if

$$E[v_1(t)(\frac{\Delta C}{C}(t) - E(\frac{\Delta C}{C}(t))] = E[v_2(t)(\frac{\Delta C}{C}(t) - E(\frac{\Delta C}{C}(t))] = 0, \qquad (8c)$$

then: $$\beta^b_{12} \equiv \frac{cov(R_j,R_M)}{var(R_M)} = \beta^a_{12} + \frac{\sigma_{12}}{\gamma^2_{22}\sigma_{cc} + \sigma_{22}}$$

Thus, the difference between β^a_{12} and β^b_{12} is just the simultaneity bias of the OLS estimator β^a_{12} in the three variable system. In that three variable system, the restrictions $\beta_{21} = \gamma_{12} = 0$ provide two sets of independent restrictions which mean that there is only one independent way of determining $R_j(t)$ and $R_m(t)$ given optimal consumption and portfolio choice.

2.2 Conditioning on the Riskless Rate

The null hypothesis in (7) is conditional on a riskless rate, R_F, which is a limitation if R_F varies over the sample period. To overcome the limitation, R_F can be added as a variable in (2), giving:

$$R_j(t) - \beta_{12}^j R_M(t) - \beta_{13}^j R_F(t) = u_1^j(t) \qquad (9a)$$

$$R_M(t) - \beta_{23} R_F(t) - \gamma_{21} - \gamma_{22} \frac{\Delta C}{C}(t) = u_2(t) \qquad (9b)$$

By definition, the riskfree rate of interest is known at the beginning of any period or "instant." In terms of the variable definitions used in the following sections for the tests, $R_F(t)$ is the rate of return on a Treasury Bill which matures with a certain payoff at the end of t. Hence, once the price of the bill is determined at the beginning of period t, $R_F(t)$ is known. The predetermination of $R_F(t)$ is basic to all the various asset pricing models which are concerned only with explanations of the risk premium which increments the riskfree rate of return to give an asset's equilibrium relative rate of return.

Should $R_F(t)$ be treated as an exogeneous variable or a predetermined endogeneous variable in (9a) and (9b)?[5] If the latter, what form should the equation for $R_F(t)$ take? Here, $R_F(t)$ is observable, and is predetermined in discrete time on the basis of a prediction of $\Delta C/C(t)$. Defining $\Delta C/C(t) \equiv \text{Predicted}[\Delta C/C(t)] + \epsilon$, and supposing, for illustrative simplicity, that R_F is a linear function of Predicted$[\Delta C/C(t)]$, an equation for $R_F(t)$ would be:

$$R_F(t) = \gamma_{31} + \gamma_{32} \frac{\Delta C}{C}(t) - \gamma_{32}\epsilon(t) \qquad (10)$$

or

$$\frac{\Delta C}{C}(t) = \gamma'_{31} + \gamma'_{32} R_F(t) + \gamma'_{32} \epsilon(t) \qquad (11)$$

where $\epsilon \perp R_F$ if $R_F(t)$ takes full advantage of the optimal prediction of $\Delta C/C(t)$. The prediction error $\epsilon(t)$ in (10) and (12) is part only of the determination of $\Delta C/C(t)$--it is not important to the determination of variables within (9) once $\Delta C/C$ is generated. Hence, $R_F(t)$ is treated as a predetermined exogeneous variable, and any "equation" for $R_F(t)$ is omitted because it would represent no more than a relation between exogeneous variables.

Application of the expectation operator conditional on $R_F(t)$ in (9) gives:

$$E[R_j(t)|R_F(t)] - \beta_{12}^j E[R_M(t)|R_F(t)] - \beta_{13}^j R_F(t) = E[u_1^j(t)|R_F(t)] \ , \qquad (12)$$

and $E[u^j(t)|R_F(t)] = 0$ in (12) when $R_F(t)$ is predetermined. Equation (12) may be rearranged as:

$$E[R_j(t)|R_F(t)] = \beta_{13}^j R_F(t) + \beta_{12}^j E[R_M(t)|R_F(t)] \ . \qquad (13)$$

The intertemporal CAPM stated in (1) is repeated, with the conditioning explicitly included as:

$$E[R_j(t)|R_F(t)] = (1 - \beta_{12}^j)R_F(t) + \beta_{12}^j E[R_M(t)|R_F(t)] \ . \qquad (14)$$

Equating (11) and (12) gives:

$$(1 - \beta_{12}^j)R_F(t) = \beta_{13}^j R_F(t)$$

$$\Rightarrow \quad H_0: \ (1 - \beta_{12}^j) = \beta_{13}^j \ . \qquad (15)$$

To obtain (12) and (9), rates of return were conditioned on the exogenously determined riskless rate $R_F(t)$. This means that (8) itself must be reparameterized so that the parameters are indeed obtained from moments conditional on $R_F(t)$. To adjust for the sample linear conditioning

of $R_M(t)$ on $R_F(t)$, we can take the residuals $\hat{V}_{M,F}$ from the regression of R_M on R_F (orthogonalization by the Gram-Schmidt procedure), and substitute in (9a) and (9b), obtaining:

$$R_j(t) - \beta^j_{12}\hat{V}_{M,F}(t) - \alpha^j_{13}R_F(t) = u^j_1(t) \tag{16a}$$

$$\hat{V}_{M,F}(t) - \alpha_{23}R_F(t) - \gamma_{21} - \gamma_{22}\frac{\Delta C}{C}(t) = u_2(t) \tag{16b}$$

$$a^j_{13} = \beta^j_{13} + \frac{\text{cov}(RM,RF)}{\text{var}(R_F)}\ \beta^j_{12}$$

$$\alpha_{23} = \beta^j_{23} + \frac{\text{cov}(R_M,R_F)}{\text{var}(R_F)}$$

Now, the coefficient β^j_{12} measures the covariance of $\tilde{R}_M(t)$ beyond its linear projection on $R_F(t)$ with the value of $\tilde{R}_j(t)$ beyond its (orthogonal) projection on $R_F(t)$. Thus, hypothesis (15) becomes:

$$\alpha^j_{13} = 1 - [1 - \frac{\text{cov}(R_M,R_F)}{\text{var}(R_F)}]\ \beta^j_{12} \tag{17}$$

2.3 Intemporal Effects: A Stochastic Beta

It might be claimed that, from simple casual observation, it is unlikely that even the term structure of expected real bond premia is constant through time as implied by the assumed constant β^j_{12}, $j=1,....N$, in (9). In the discrete-time tests formulated here, a time varying β^j_{12} of the following form is allowed:

$$\beta^j_{12} = \beta^j_{12,0} + \beta^j_{12,1}\ R_F(t) + \beta^j_{12,2}\Delta R_F(t) \tag{18}$$

Thus, bond betas, and hence the shape of the yield curve, can vary up to a linear approximation with the level and first differences of the nominal

short-term (here one month) interest rate. To the extent that the one month rate is a predictor of inflation rates, $\Delta R_F(t)$ is the realized change in expected inflation rates. To be entirely rigorous in formulating (18), one would need to be able to derive a closed-form solution for equilibrium long-term bond returns as a function of short rates, changes in short rates, and short-rate dynamics. (Of course, if that closed-form solution were available, one could substitute out the "reduced form" beta coefficients and estimate the equilibrium bond pricing model directly.) Note that (17) implies, consistent with the expected return model above, that the beta is a deterministic function of variables known at the beginning of (discrete) interval t. Substituting (18) into (9):

$$R_j(t) - \beta^j_{12,0} R_M(t) - \beta^j_{12,1} R_F(t)R_M(t) - \beta^j_{12,2}\Delta R_F(t)R_M(t) - \beta^j_{13} R_F(t) = u^j_1(t) \tag{19a}$$

$$R_M(t) - \beta_{23} R_F(t) - \gamma_{21} - \gamma_{22} \frac{\Delta C}{C}(t) = u_2(t) \tag{19b}$$

At least two assumptions in (19b) are immediately apparent: (i) "the market" consumption beta is assumed to be unaffected by the interst rate dynamics; and (ii) since there is a nonconstant exogenous variable $\frac{\Delta C}{C}(t)$ in (19), identification is still possible with the cross-product terms arising from the time-varying beta (Fisher [15]).

In line with the earlier discussion, the tests will involve a two-step procedure. First, are $\beta^j_{12,1}$, and $\beta^j_{12,2}$ significantly different from zero? If so, is the restriction (15) or (17) met by the single period model?

3. Inflation and a Second "Utility" Index

The consumption model (1) and the corresponding test system (9) allow for price level changes only insofar as they are impounded in the one-dimensional consumption index. Yet, the utility the investor derives from the proceeds of a default-free bond is uncertain not only because of shifts, over the bond's term to maturity, in the aggregate real stock of productive capital and technological changes, but also because of shifts in the price index at which the nominal proceeds can be converted into real goods. Thus, even if the technological changes are not taken into account, the investor will still face a two-dimensional uncertainty with respect to real wealth changes and price index changes.[6] The two sources of uncertainty cannot, in general, be adequately captured by (say) one nominal consumption index.[7] Therefore, tests of the hypothesis that the consumption CAPM performs better than the traditional CAPM, as above, might be rendered invalid by the failure to account adequately for stochastic price level changes. This possibility would be reinforced if, as Fama [12] concludes, the nominal factors are predominant in the determination of the nominal term premium.

When the general price level P is explicitly taken into account in investment-consumption decisions, (1) is replaced by the following model:

$$E(R_j) - R_F - \mathrm{cov}(R_j,\pi) = \beta_j[E(R_M) - R_F - \mathrm{cov}(R_M,\pi) \tag{20}$$

where: π = the inflation rate

$$\beta_j = \frac{\mathrm{cov}(R_j, \frac{\Delta C^*}{C^*})}{\mathrm{cov}(R_M, \frac{\Delta C^*}{C^*})}$$

$C^*(t)$ = per capita real consumption, or unspecified marginal utility surrogate, as a function of time.

Inflation, i.e., π, enters (20) in two ways: First, since inflation is correlated (of order Δt) with R_j and R_M and in addition is an argument in the function for optimal real capita consumption, it enters β_j in (20) as a component of both the numerator and denominator. The importance of the inflation component of the "real consumption" beta in (20) depends upon the extent to which variables such as real per capita consumption are not homogeneous of degree zero in the price level, i.e., the extent to which the Phillips curve is not vertical. Second, it enters in a purely "nominal" way. That is, to the extent that an asset's nominal returns are correlated with price changes, and hence with nominal wealth, those returns yield the change in numeraire required to purchase real consumption goods or units of real wealth. Since the asset's nominal payoffs are correlated with price changes which proportionately impact both nominal consumption expenditures and nominal wealth, the coefficients of $\text{cov}(R_j,\pi)$ and $\text{cov}(R_M,\pi)$ in (20) are unity because there is no alteration in the ratio of real wealth (expected future consumption) to present consumption, so that risk aversion is not relevant.

To develop a test of (20) applying to nominal rates of return, the original model (2) needs to be replaced by:

$$[R_{jt} - R'_{jt}(\pi_t - E(\pi_t|\pi_{t-1},\ldots))] - \beta_{12}[R_{Mt} - R'_{Mt}(\pi_t - E(\pi_t|\pi_{t-1},\ldots))] - \alpha'_j = u^j_{1t} \qquad (21a)$$

$$[R_{Mt} - R'_{Mt}(\pi_t - E(\pi_t|\pi_{t-1},\ldots))] - \gamma_{21}\gamma_{22}\frac{\Delta C^*}{C^*} = u_{2t} \qquad (21b)$$

where: $R'_{jt} \equiv R_{jt} - E(R_{jt}|R_{ft},\pi_{t-1},\ldots)$, $R'_{Mt} \equiv R_{Mt} - E(R_{Mt}|R_{ft},\pi_{t-1},\ldots)$.

To verify that (21), with parameter restrictions, reduces to (20), expand (21a) and take expectations conditional on $R_{ft},\pi_{t-1},\ldots$::

Taking unconditional expectations:

$$E(R_{jt})-E[R'_{jt}-E(R'_j)][\pi_t-E(\pi_t|\pi_{t-1},\ldots)]-\beta_{12}\{E(R_{Mt})$$
$$-E(R'_{Mt}-E(R'_{Mt})][\pi_t-E(\pi_t|\pi_{t-1},\ldots)\} = \alpha'_j$$

which, with fully explicit conditioning, is

$$E(R_{jt}|R_{Ft},\pi_{t-1},\ldots)-\text{cov}(R_j,\pi_t|R_{Ft},\pi_{t-1},\ldots)$$
$$-\beta_{12}\{E(R_{Mt}|R_{Ft},\pi_{t-1},\ldots) \qquad (22)$$
$$-\text{cov}(R_{Mt},\pi_t|R_{Ft},\pi_{t-1},\ldots)\} = \alpha'_j$$

which will be (20) with the restriction:

$$\alpha'_j = R_F(1-\beta_{12}) \qquad (23)$$

provided that $\beta_j = \beta_{12}$. By substituting (21b) in (21a):

$$\beta_{12} = \frac{\text{cov}(R_{jt} - R'_{jt}(\pi_t-E(\pi_t|\pi_{t-1},\ldots)),\ \frac{\Delta C^*}{C^*})}{\text{cov}(R_M - R'_M(\pi_t - E(\pi_t|\pi_{t-1},\ldots)),\ \frac{\Delta C^*}{C^*})}$$

$$= \frac{\text{cov}(R_j,\frac{\Delta C^*}{C^*})-E(R'_j)\text{cov}[\pi_t-E(\pi_t|\pi_{t-1},\ldots),\frac{\Delta C^*}{C^*}] - E[\pi_t - E(\pi_t|\bullet)]\ \text{cov}(R'_j,\ \frac{\Delta C^*}{C^*})}{\text{cov}(R_M,\frac{\Delta C^*}{C^*})-E(R'_M)\text{cov}[\pi_t-E(\pi_t|\pi_{t-1},\ldots),\frac{\Delta C^*}{C^*}] - E[\pi_t - E(\pi_t|\bullet)]\ \text{cov}(R'_m,\ \frac{\Delta C^*}{C^*})}$$

$$= \beta_j \quad \text{since} \quad E(R'_j) = E(R'_M) = E[\pi_t - E(\pi_t|\bullet)] = 0 \quad .$$

That is, conditional on $R_F(t)$ and $E(\pi_t|\pi_{t-1},\ldots)$, (20) implies the restriction (23) across the coefficients of (21). In words, given the statistical model in (21), the consumption ICAPM in (20) implies the constraint (23) on the coefficients of (21). If instead the market model equation (21a) is considered alone, the traditional CAPM implies constraint (23). Note that a non-zero $\text{cov}([\pi_t - E(\pi_t|\bullet)],\ \Delta C^*/C^*)$ in (24), which would capture any real sector-monetary sector interdependence arising in equilibrium models like those of Lucas [32,33], Sargent [46,47], does not directly enter the tests.

There will be measurement errors in the endogenous variables in (21) if $E(\pi_t|\cdot)$, $E(R_m|\cdot)$, and $E(R,|\cdot)$ are replaced by their sample estimates --for example, if inflation were a random walk, $(\pi_t - E(\pi_t|\cdot)]$ would be realized first differences of inflation rates. But, as Hausman [22] notes, in the simultaneous equations context, "consideration of errors... in the endogenous variables is not interesting since they are observationally equivalent to the errors in the equations." (p.392).

4. Unobserved Marginal Utility or Per Capita Consumption

Heretofore, discussion has centered on the consumption form of the ICAPM. However, as explained in the introduction, the idea behind the consumption model is that per capita consumption be regarded as an instrument for the typical investor's marginal utility. But if per capita consumption is itself measured with error in the available data, measured consumption might more appropriately be regarded as an indicator, along with other variables such as per capita production or optimum portfolio allocations, of a latent variable directly quantifying the average investor's marginal utility. Let $\xi(t)$ be this latent variable. Then, in the tests, the measurement error model will be:

$$X_1 = \lambda_1\xi(t) + \varepsilon_1(t)$$

$$\vdots \qquad (25)$$

$$X_k = \lambda_k\xi(t) + \varepsilon_k(t)$$

where $\xi(t) \equiv \Delta C/C(t)$ in all of the previous discussion, any one of the indicator variable X's might be observed per capita consumption, and the properties of $\underline{\varepsilon}' = [\varepsilon_1,\ldots,\varepsilon_k]$ will be discussed below.

Note that simply plugging any of the X's in (20) into (8) or (16) as an instrument for $\xi(t)$ or $\frac{\Delta C}{C}(t)$ will not induce an errors-in-variables inconsistency in β_{12} since the unobservable variable does not appear in (8a) or (16a). The error will cause inconsistent estimates of γ_{21}, γ_{22}. Given the measurement error model (20) though, the estimates of β_{12} will be inefficient, as would be the tests below which are based on the covariance matrix of disturbances in (8) and (16).

Adding specification (25) to (21) and forming a multivariate system gives:

$$R_a^1(t) - \beta_{12}^1 R_{Ma}(t) - \beta_{13}^1 R_F(t) = u_1^1(t) \qquad (26a)$$

$$R_a^2(t) - \beta_{12}^2 R_{Ma}(t) - \beta_{13}^2 R_F(t) = u_1^2(t)$$

$$\vdots$$

$$R_a^N(t) - \beta_{12}^N R_{Ma}(t) - \beta_{13}^N R_F(t) = u_1^N(t)$$

$$R_{Ma}(t) - \beta_{23} R_F(t) - \lambda_{21} - \lambda_{22}\xi(t) = u_2(t) \qquad (26b)$$

$$\begin{aligned} X_1(t) &- \lambda_1 \xi(t) = \varepsilon_1(t) \\ &\vdots \\ X_k(t) &- \lambda_k \xi(t) = \varepsilon_k(t) \end{aligned} \qquad (26c)$$

where: $R_a^j(t) \equiv R_j(t) - R_j'(t)(\pi(t) - E[\pi(t)|\pi(t-1)....])$ and similarly for $R_{Ma}(t)$, and $R_j'(t)$ is defined below (21). When appropriate, (26a) and (26b) are reparameterized as in (16) and the stochastic beta model (18) is introduced.

The linear structure (26) might be recognized as a special application of the general multiple indicator, multiple cause (MIMIC) model, discussed by Zellner [51], Goldberger [18,19] and Aigner [1].[8] If the elements of $\underline{\varepsilon}(t)$ were mutually independent, (26) is essentially a (confirmatory) factor analysis model, with $[\gamma_1,\ldots,\gamma_k]$ interpretable as factor loadings.

In the analysis, five indicator variables X_j, $i = 1,\ldots,5$ described in the next section, are used: seasonally adjusted retail sales of nondurable consumer goods, industrial production of nondurable consumer goods, the average realized real rate of return on one-month T-Bills over the past twelve months, the growth rate in the monetary base over the past twelve months, and an interpolated monthly series of quarterly per capita consumption estimates based on those reported in the Survey of Current Business. In the MIMIC model framework, these five variables are being treated as indicators of the true well-offness quantity rather than causes of that quantity. While these variables, as either indicators or causes,[9] could be regarded as instruments for $\xi(t)$ or $\Delta C/C(t)$, their interpretation as indicators here seems more in line with the notion that $\xi(t)$ or $\Delta C/C(t)$ is inherently unobservable, rather than directly observable, albeit with error, and with the specification that these X_j, $i = 1,\ldots,5$ are, at most, jointly stochastic but channelled through a common unobserved variable $\xi(t)$ rather than generating or causing $\xi(t)$.

In (26) the vector $\underline{u}' = [u_1^1,\ldots,u_1^N,u_2]$ is assumed orthogonal to $\underline{\varepsilon}$. This assumption is an identifying one and hence non-testable. However, it does have some theoretical support.

Interpreting $\xi(t)$ as a marginal opportunity or aggregative well offness variable, the deviation of $R_M(t) - \beta_{23}R_{Ft}$ from the linear function of $\xi(t)$: $\gamma_{21} + \gamma_{22}\xi(t)$, could be regarded analogously to transitory wealth. If (say) $\varepsilon_1 = X_1 - \lambda_1\xi(t)$ is a measure of transitory consumption ("measurement error" in a slightly different sense), then the orthogonality of u_2 and ε_1 is consistent with the permanent income literature. Similar arguments can be made about u_1 and ε_1, ε_2, ε_3, ε_4, ε_5.

It may be verified that (26) will not be identified without a normalization on the $\underline{\lambda}$ coefficients, (or the variance of an element of $\underline{\varepsilon}$). Here, we set $\lambda_1 \equiv 1$. Further, the exogeneous variables $(\xi, R_F, X_2, X_3, X_4, X_5)$ are assumed to be jointly distributed, and the measurement error $\underline{\varepsilon}$ in (26) to be orthogonal to these variables (as well as to ξ). More generally than if (25) were a factor model, elements of $\underline{\varepsilon}$ may be correlated, e.g., in the multivariate marginal distribution of (X_1, X_2), $\text{cov}(X_1, X_2) = \gamma_2\gamma_3\sigma_{\xi\xi} + \text{cov}(\varepsilon_1, \varepsilon_2)$.

As pointed out by Hausman [22], if $(\xi, R_F, X_2, X_3, X_4, X_5)$ are multivariate normal,

$$\underline{\xi} = Z\underline{\alpha} + \underline{v} \tag{27}$$

where $Z \equiv [\underline{1}, \underline{X}_2, \underline{X}_3, X_4, X_5, \underline{R}_F]$ and $v \perp Z$. Substituting (27) into (26a) and (26b), (26) becomes:

$$
\begin{aligned}
R_a^1(t) - \beta_{12}^1 R_{Ma}(t) - \beta_{13}^1 R_F(t) &= u_1^1(t) \\
&\vdots \\
R_a^N(t) - \beta_{12}^N R_{Ma}(t) - \beta_{13}^N R_F(t) &= u_t^N(t) \qquad (28) \\
R_{Ma}(t) - \beta_{23} R_F(t) \qquad \gamma_{21} - Z(t)\gamma_{22}\,\underline{\alpha} &= u_2(t) + \gamma_{22}\, v(t) \\
X_1(t) \quad - Z(t)\underline{\alpha} \quad &= v(t) \quad + \varepsilon_1(t) \\
\vdots \qquad\qquad \vdots \qquad\quad \vdots \qquad \vdots &
\end{aligned}
$$

In this instrumental variable procedure, the restriction $\mathrm{cov}(u_1^j, u_2) = 0$, for all j, implied by the null hypothesis that the consumption CAPM (1) does not "overfit" the traditional CAPM, can be tested by using the procedure in Hausman [23]. There he proposes a test of exogeneity in (21a) based on the difference between an OLS estimator of β_{12} in (21a), which is consistent and efficient under the null hypothesis $\mathrm{cov}(u_1, u_2) \neq 0$, but inconsistent under the alternative $\mathrm{cov}(u_1, u_2) \neq 0$, and (here) an IV estimator which is consistent under both the null and alternative, but efficient only under the former. The formulation of the test used here involves OLS applied to the following transformation of (26a):

$$R_a^j(t) = \beta_{12}^j \hat{R}_{Ma}(t) + \beta_{13}^j R_F(t) + \phi^j v(t) + u_j^1(t) \qquad (27)$$

where: $v(t) \equiv R_{Ma}(t) - \hat{R}_{Ma}(t)$, and $\hat{R}_{Ma}(t)$ is the instrumental variable estimator obtained from (26b) and (26c). The standard F test for $\phi = 0$ is asymptotically equivalent to the conventional likelihood ratio, LM and Wald procedures for testing whether $\sigma_{12} = 0$.[12]

Given the assumptions on $\underline{u}$ and $\underline{e}$, (28) contains restrictions on the disturbance matrix and on the slope coefficients. Thus, although a full

instrumental variable method (e.g., 3SLS) will be consistent, it will not be fully efficient (Rothenberg and Leenders [42]). Fully efficient estimates and tests are obtained here using the program MOMENTS[10] to obtain FIML estimates of (26). In several cases, FIML estimates have also been obtained from the program LISREL IV[11] for comparison. Tests are likelihood ratio tests using these FIML programs.

5. Data and Results

The cross-section of bond returns used in (28) is taken from the Center for Research in Security Prices (CRSP) Government Bond Returns File. Since a time series of monthly returns across the maturity spectrum cannot be constructed from the returns on single bonds, a cross-section of portfolios composed of bonds within given ranges of maturity has been constructed.[13] The following cross-section of six portfolios has been used here:

Portfolio Number	Maturity	Average Maturity 1958/2 - 1978/12
1	0 - 6 months	2.70 months
2	6 months - 1 year	8.74 months
3	1 year - 2 years	1.47 years
4	2 years - 3 years	2.48 years
5	3 years - 5 years	3.41 years
6	> 5 years	8.74 years

As mentioned earlier, five indicator variables ($\underline{X}(t)$ in (25)) are used: seasonally adjusted real retail sales of nondurable consumer goods, industrial production of nondurable consumer goods, the average realized real rate of return on one-month T-Bills over the past twelve months, the growth rate in the monetary base over the past twelve months, and the interpolated monthly series of quarterly per capita consumption obtained from the MPS Quarterly Econometric Model data file. "The" market returns are those on the value-weighted NYSE index, but it has been verified that the results here hold _a fortiori_ if the equally weighted index is used.

For convenience, the five hypotheses of interest are restated here (the parameters referenced in the first four are in (26) and the fifth in (18):

- - - - - - - - - - - - - - -

(1) $H_0(1)$: $(1 - \beta_{12}^j) = \beta_{13}^j$, $j = 1, \ldots, 6$; $\mathrm{cov}(u_1^j, u_2)$ unrestricted

$H_A(1)$: $(1 - \beta_{12}^j) \neq \beta_{13}^j$, $j = 1, \ldots, 6$; $\mathrm{cov}(u_1^j, u_2)$ unrestricted

The null hypothesis $H_o(1)$ is that the consumption ICAPM (with constant coefficients) holds.

- - - - - - - - - - - - - - - - -

(2) $H_0(2)$: $(1 - \beta_{12}^j) = \beta_{13}^j$, $j = 1, \ldots, 6$; conditional on $\mathrm{cov}(u_1^j, u_2) = 0$

$H_A(2)$: $(1 - \beta_{12}^j) \neq \beta_{13}^j$, $j = 1, \ldots, 6$; conditional on $\mathrm{cov}(u_1^j, u_2) = 0$

The null hypothesis is that the traditional CAPM (with constant coefficients) holds.

- - - - - - - - - - - - - -

(3) $H_0(3)$: $\text{cov}(u_1^j, u_2) = 0$; for any β_{12}^j, β_{13}^j, $j = 1, \ldots, 6$; $\underline{\beta}$ unrestricted

$H_A(3)$: $\text{cov}(u_1^j, u_2) \neq 0$; for any β_{12}^j, β_{13}^j, $j = 1, \ldots, 6$; $\underline{\beta}$ unrestricted

The general one-period model with a utility instrument (e.g., true per capita consumption) reduces to the traditional single-period model; i.e., the utility instrument does not "improve" upon the market index as a surrogate for wealth.

- - - - - - - - - - - - - -

(4) $H_0(4)$: $\text{cov}(u_1^j, u_2) = 0$; $\beta_{13}^j = (1 - \beta_{12}^j)$, $j = 1, \ldots, 6$

$H_A(4)$: $\text{cov}(u_1^j, u_2) \neq 0$; $\beta_{13}^j = (1 - \beta_{12}^j)$, $j = 1, \ldots, 6$

The consumption ICAPM "reduces" to the traditional CAPM.

- - - - - - - - - - - - - -

(5) $H_0(5)$: $\beta_{12,1}^j = \beta_{12,2}^j = 0$, $j=1,\ldots.6$; $\text{cov}(u_1^j, u_2)$ unrestricted

$H_0(5)$: $\beta_{12,1}^i \neq 0$ and/or $\beta_{12,2}^j \neq 0$; $\text{cov}(u_1^j, u_2)$ unrestricted.

The null hypothesis is that, conditional on the varying beta model used, the beta is constant. If $\text{cov}(u_1^j, u_2)=0$, $j=1,\ldots,6$, the null would be a test of constancy of coefficients in the traditional CAPM.

- - - - - - - - - - - - - -

Table 1(a) contains the $\hat{\beta}_{12}$ and $\hat{\beta}_{13}$ coefficients in the Hausman exogeneity test of the value-weighted rate of return on the market R_{Mt} in (26), over the period March 1959 to December 1978, for the six bond portfolios with increasing terms to maturity which were described at the beginning of this section. The $\hat{\beta}_{12}$ coefficient of the instrumental variable for R_{Mt} increases monotonically until bond 4 (which has a maturity of two to three years), and then decreases. That is, if the real term structure is constant over that period, it is "humped." Such a cross-sectional pattern would be consistent with the longest term bonds having a "depression insurance" value (presumably the peak at mid-range maturities would imply something about the expected length of depressions under that explanation). The absolute values of the point estimates of the β_{12} coefficients are small relative to those usually seen in the stock market where betas average to one when the rate of return on the NYSE index is used as the market rate of return.

There are several variations of the Hausman test (see Nakamura and Nakamura [38] and Holly and Monfort [27]), and the one used here involves testing whether $\beta_{12}^{j} = \phi_{12}^{j}$ in (27) across all j=1,...,6 bonds. The point estimates appear to diverge most at the short end of the maturity spectrum while at the long end, $\hat{\beta}_{12}^{6} = 0.072$ and $\hat{\phi}_{12}^{6} = 0.079$ are almost identical (bond portfolio 6 contains bonds with maturities greater than five years; it has an average maturity of 8.74 years). The close agreement for long maturities is interesting because it is most likely that any data problems associated with nontrading of bonds would have greatest impact there.

The estimates of β_{13}, the coefficient of the one-month T-Bill (nominally riskless) rate follows the cross-sectional pattern which would be

predicted by null hypothesis (1) above, though it apears too large in point value at the short maturities.

Table 1(b) provides the same Full Information Maximum Likelihood (FIML) estimates provided by the MOMENTS and LISREL IV programs when there are no restrictions placed on the covariance matrix of the disturbances in (26). The MOMENTS $\hat{\beta}_{12}$ is almost identical to the instrumental variable coefficient in Table 1(a) for the shortest maturity, but is smaller in magnitude as maturity increases and "peaks" at maturity three, not maturity four as in Table 1(a). Interestingly, the $\hat{\beta}_{13}$ values fall more quickly than those in Table 1(a) as maturity increases, and do not turn back up for the long-term maturities. The LISREL IV estimates of both β_{12} and β_{13} are substantially greater than the instrumental variable and MOMENTS estimates for the three longest maturities and serve to illustrate some of the apparent convergence problems which the algorithm experiences. Table 1(b) also gives the analogous FIML estimate for the case where $\text{cov}(u_1^j, u_2)$ is restricted to zero for all $j=1,\ldots,6$ in (26). This time the MOMENTS estimates agree quite closely with the instrumental variable estimates for β_{12}, while LISREL IV generates substantially smaller point estimates at the short maturities.

In Table 2(a), the test statistics for $H_o(3)$ (that the constant "consumption" beta model reduces to the Sharpe-Lintner CAPM which uses R_M as a wealth surrogate), suggest that the hypothesis can be rejected at conventional significance levels. In the case of the Hausman test, the statistic used is the Wald statistic for testing the equality of $\underset{\sim}{\beta}_{12}$ and $\underset{\sim}{\phi}_{12}$ in Tables 1(a) and 3(1) (below). In the FIML tests, χ^2 is $-2 \ln\lambda$, where λ is the usual likelihood ratio value.[14] The large

value of the MOMENTS test value reflects a failure in this case by that algorithm to find a step size which would improve upon the likelihood function when the disturbance covariance matrix in (26) was restricted in accord with $H_o(3)$.

It was pointed out in Section 2.2 that the asset pricing restrictions on the β_{12} and β_{13} coefficients in each equation should actually be tested on the conditional counterparts of those moments in (26). In (16), (26a) was reparameterized by replacing the market return R_{Ma} by its deviation about the expected value of that market return conditional upon the one-month T-Bill rate. In Tables 3(a) and 3(b), model (26) has been re-estimated using that conditionally unexpected market rate of return variable. The FIML results have been recomputed only with MOMENTS because of the expense involved. The point estimates of the coefficient of the orthogonalized instrumental market return in Table 3 are higher for medium maturities and lower at longer maturities, while the FIML estimates of β_{12} are close to those of Table 1(b) where the original market return variable was used. This time, the p-value for rejection of hypotheses $H_o(3)$ is approximately 0.025 for the Hausman test of $\text{cov}(u_1^j, u_2)=0$ in (26), while for the FIML test, the p-value is less than 0.001 (the FIML converged for both the restricted and unrestricted disturbance matrix).

When the slope coefficients β_{12} and β_{13} are restricted in accordance with the null hypothesis $H_o(4)$ above (i.e., restrictions (15) and (17) are imposed on those coefficients before the hypothesis $\text{cov}(u_1^j, u_2)=0$, $j=1,\ldots,6$ in (26) is tested), the Hausman and FIML statistics for that covariance restriction are 24.74 and 1074.15 respectively, when the orthogonalized market return variable is used. Thus, the conclusions for $H_o(4)$ are effectively similar to those for $H_o(3)$.

The results of the last paragraph suggest that the slope coefficient restrictions $H_o(1)$ and $H_o(2)$ do not seem to affect the exogeneity results much. In Table 4, the F-statistics are given equation-by-equation for testing the restriction $H_o(2)$ in the instrumental variable regression and for testing $H_o(1)$ in the least-squares regression case. The slope coefficient restrictions are implied by the CAPM when the NYSE market index is an adequate descriptor of wealth (the traditional Sharpe-Lintner CAPM) and when a consumption-type surrogate for optimally composed wealth is used (a Breeden-type CAPM). Results are also given in Table 4 for tests of the appropriate coefficient restriction (17) when the orthogonalized market return variable is used. Irrespective of whether the restriction (15) or (17) is tested, the null hypothesis that the asset pricing models hold is rejected at conventional sampling theory levels[16] only for the shortest maturity bonds (less than six months to maturity), and at the 5% level for the next-to-shortest maturity bonds (maturities of six to twelve months). In Table 2(a), the likelihood ratio test based on LISREL IV appears to reject $H_o(1)$ and $H_o(2)$, as does the MOMENTS result for $H_o(2)$. In testing $H_o(1)$ in the Table 2(a) case and both $H_o(1)$ and $H_o(2)$ in the Table 2(b) case, the FIML-MOMENTS algorithm actually converged to a higher point of the likelihood function in the restricted rather than unrestricted case, from which we would logically conclude either that the convergence criterion is not sufficiently sensitive to the test, or that the difference in likelihoods is negligible, i.e., "there is not much in it." With the exception of the shortest maturity portfolio, it could be concluded that the rejection of the traditional Sharpe-Lintner CAPM estimated using the NYSE stock index is primarily due to inadequacy of that index rather than to failure of the slope coefficient restrictions implied by that pricing

model. In addition, the test of the latter rstrictions does not seem to depend heavily upon the conditioning of the market model coefficients on the riskless rate.

Finally, we examine whether there are state variables other than wealth which would cause the bond "betas" β_{12}^{j}, j=1,...,6 (or real term structure) to vary through time. It will be recalled from the discussion in Section 2.3 that advantage was taken of the extant term structure literature in proposing that the variation would be functionally related to the short-term (one month) riskless rate and to the prior month change in that rate. Table 5 presents the F-tests for the null hypothesis $H_o(5)$ that the variation in real betas is zero for the instrumental variable ("consumption beta") coefficient and ordinary least-squares (traditional CAPM beta) cases. The null hypothesis is not rejected in Table 5 at any maturity. For the traditional CAPM, the likelihood ratio χ^2 test statistics for beta constancy are 666.32 and 649.00 for the market return and orthogonalized market return variables, respectively. For the consumption-type beta case, the algorithm converged to a positive value for the log likelihood in the market return case and to a higher likelihood value for the restricted than unrestricted model in the orthogonalized market return case. It would seem that we could only reject beta constancy if convergence is deemed acceptable in one of these cases even though it is obviously not in the other.

The performance of the FIML algorithms in the varying beta case is interesting from another point of view. In some earlier cases where the beta was assumed constant, the FIML algorithms had convergence problems. One possible reason might have been traceable to systematic sources of uncertainty which, with a constant beta, "ended up" in the disturbance matrix. If those common elements were important relative to the

idiosyncratic variances, the disturbance matrix could be close to singular, and hence the model nearly underidentified. However, "extracting" common components from the disturbance matrix with the varying beta model does not seem to help much, which adds further to the suggestion that there are not multiple sources of systematic uncertainty.

The conclusion in Table 5 might, of course, simply reflect the inadequacy of the varying beta model in the alternative hypothesis $H_A(5)$, and perhaps further work would be profitable in this regard at the anlytical rather than empirical level. However, conditional on current theoretical and empirical term structure knowledge, it is hard to think of a more plausible model.

6. Summary and Conclusions

The goal of this paper is the development of a methodology for assessing the incremental explanatory power of asset pricing models which elaborate upon the traditional Sharpe-Lintner CAPM. It was shown that progression from a stable coefficient consumption CAPM to the traditional CAPM imposes increasingly severe restrictions upon a simultaneous equation model which can be interpreted as the "empirical counterpart" of the consumption CAPM in the same way that the market model forms the "empirical counterpart" of the traditional CAPM. The approach is an application of a very general one in economics in which theoretical models imply testable (overidentifying) restrictions on reduced-form models fitted to the data.

Tests have been implemented on government bond returns. The results offer little support for the use of the usual value-weighted index of stock returns when estimating "constant beta" asset pricing models applied to Government bond returns. When inflation is properly accounted for in the consumption- type CAPM, the restrictions necessary to reduce it to the form of the traditional two- period CAPM are rejected. Although the results are not reported here, it turns out that if instead inflation is accounted for by use of a single index of nominal per capita consumption, in the case of the consumption CAPM, or the nominal stock market index in the case of the traditional CAPM, neither model fits very well. This is an important conclusion for it suggests, at least insofar as bond returns are concerned, that nominal price changes cannot be finessed as is typically done in the statement and testing of CAPM models. That is, the fact that asset pricing models should be expressed in real rates of return cannot be ignored.

Second, for all but the shortest maturity bonds (less than one year), the asset pricing restrictions imposed by the traditional Sharpe-Lintner CAPM or by a Breeden-type CAPM cannot be rejected. In particular, the restrictions cannot be rejected for the longest maturity bonds (maturity of more than five years) for which the returns series look most like that of common stocks. By contrast, Gibbons [17] found that similar restrictions imposed by the Black form of the traditional Sharpe-Lintner model could be rejected for common stock returns. We can conjecture why this is so. First, the riskless rate in the Sharpe-Lintner model is included in the tests here and allowed to vary through time. In the Gibbons tests, the expected rate of return on the zero-beta asset which, in the Black model, replaces the riskless asset, was assumed constant over time. Here, only the bond expected rates of return conditional on the one-month T-Bill rate must be assumed constant. (Of course, the offset in the tests here is that the traditional Sharpe-Lintner CAPM may be more likely to be rejected in favor of a Breeden-type model because the riskless rate does not correspond with the use of the NYSE stock index as a good surrogate for "true wealth" (Roll [40])). Second, it is possible that the power of the test here is low, just as is the power of tests of different models of the behavior of expected real rates of interest. Yet one would expect a priori that the power would be most lacking at the short end of the maturity spectrum where the restriction is actually rejected. Third, there could be some other sort of misspecification here that afflicts the longer-term equations more severely.

Clearly, however, considerable work remains before all issues are finally resolved. First and foremost is the ever-present possibility that the statistical assumptions of the tests may be violated despite best

efforts at diagnostic checking. Whilst the tests assume only that conditional expected bond returns are constant, (or if variable, that they accord with the varying beta model in (19)), the variance-covariance matrix of the deviations of those returns from their conditional expectations is assumed constant. Second, there is always reason to be careful with regressions involving variables like the short-maturity bond returns here that are not covariance stationary. There appears to be a slight realized first-order autocorrelation of the disturbances in (26) in the 1960's, but it is difficult to draw firm conclusions (particularly since these auto-correlations are not independent). Finally, all the tests are asymptotic.

Second, tax effects have been ignored in the tests. It can be proved (Marsh [34], Appendix F) that fairly general taxation policies have no effect on the prices of discount bonds within the consumption CAPM. For coupon bonds, however, the absence of tax effects must rely on more stringent assumptions (say) within a Miller-Scholes [37] type world. Also, the government bonds are not treated as net wealth, one consequence being that they are not added to the stock market index as a surrogate for wealth even though the pricing model is being applied to bonds. Some inter-generational models associate equilibrium effects with the issuance of bonds, though the existence of these effects often requires restrictions on alternative modes of inter-generational transfer (see Samuelson [43], Barro [3], and Drazen [11]).

Finally, the estimation and test procedures implemented here take cognizance of the issues raised by Roll [40]. Before explaining, however, note that the methodology here is not that suggested by Roll [41]. Roll's [41] reply to Mayers and Rice emphasizes that "the only relevant question is whether the CAPM is exactly linear (in beta)" (p.395) and that the

alternative is a model with "an unspecified but nonconstant" intercept term. The tests here can be expressed in that form, but the methodology phrases the alternative partly in terms of the CAPM beta itself--the β_{jI} in Rolls' terminology. Stochastic state variables enter through a stochastic β_{jI}.

The central problem for CAPM tests raised by Roll [41, p. 395] is outlined as follows: In every sample, there will always exist portfolios that could serve as market index proxies guaranteeing that:

$$E(R_j) = \gamma_0 + \gamma_1 \beta_{jI} \quad ,$$

where $E(R_j)$ is the expected rate of return on individual asset j and β_{jI} is its "risk" coefficient for a given index, I. In the same sample, there will always be proxies which guarantee that

$$E(R_j) = \gamma_0 + \gamma_1 \beta_{jI} + \alpha_{jI} \quad ,$$

where α_{jI} is an unspecified but nonconstant vector (which is, incidentally, different for different choices of the index I).

Empirical tests of the CAPM thus face a dilemma. If $\alpha_{jI} \equiv 0$ and the CAPM is accepted, it may be that although the true market index is not efficient, the proxy is: in fact, the existence of mean-variance efficient proxies is guaranteed. On the other hand, if $\alpha_{jI} \equiv 0$ and the CAPM is rejected, the opposite may have occurred; i.e., the proxy used in the test may be ex post inefficient, so that the test says nothing about the validity of the CAPM if the "true" market index is properly used.

The test approach used here makes explicit allowance for deviations of measured wealth from the true descriptor of marginal utility. Various proxies are treated as indicators of the "true wealth" variable; none are required to individually be the true wealth figure.

The approach in no way contradicts Roll's central theme. The errors-in-the-variables model used here, complete with identification assumptions, is simply a sophisticated way of describing the difference between true per capita wealth and its measured counterpart. The model hypothesis is a joint hypothesis of the validity of the errors-in-the-variables model and the asset pricing model (1).

Table 1(a)

Estimates of the coefficients of the Hausman two-step instrumental variable procedure for testing exogeneity of the value-weighted NYSE market rate of return in explaining long-term bond returns[1/] over the period March 1959 to December 1980.

$$\text{MODEL:} \quad R_a^j(t) = \beta_{12}^j \hat{R}_{ma}(t) + \beta_{13}^j R_F(t) + \phi_{12}^j v(t) + u_j(t) \qquad (27)$$

$$\hat{R}_{ma}(t) = \underline{\gamma}' \underline{Z}(t) \quad [2/]$$

$$v(t) = R_{ma}(t) - \hat{R}_{ma}(t)$$

Bond Portfolio	$\hat{\beta}_{13}$ (Std. Error)	$\hat{\beta}_{12}$ (Std. Error)	$\hat{\phi}_{12}$ (Std. Error)	DW[3/]	R^2
1	1.080 (0.017)	0.024 (0.067)	0.002 (0.001)	1.945	0.952
2	1.075 (0.046)	0.050 (0.017)	0.015 (0.004)	1.847	0.751
3	1.000 (0.079)	0.080 (0.030)	0.030 (0.074)	1.840	0.051
4	0.963 (0.116)	0.089 (0.044)	0.045 (0.011)	1.903	0.328
5	0.880 (0.152)	0.086 (0.058)	0.060 (0.014)	1.914	0.218
6	0.897 (0.212)	0.072 (0.081)	0.079 (0.019)	2.033	0.145

1/ $R_a^j(t) \equiv R^j(t) - [R^j(t) - E(R^j(t) | R_F, \pi(t-1), \ldots)][\pi(t) - E(\pi(t) | R_F, \pi(t-1), \ldots)]$ where $R^j(t)$ is the rate of return on a portfolio of bonds with maturity range j, $\pi(t)$ is the rate of inflation (consumer price index), and R_F is the (known) nominal rate of return on a T-bill with one month to maturity.

2/ The indicator variables: seasonally adjusted retail sales of nondurable consumer goods, industrial production of nondurable consumer goods, the average realized real rate of return on one-month T-bills over the past twelve months, the growth rate of the monetary base over the past twelve months, and an interpolated monthly series of quarterly per capital

2/ (Continued)
consumption, in addition to the exogeneous nominal one month T-bill return comprise the vector $\underline{Z}$ in these regressions.

3/ With three regressors, the 0.05 significance level lower and upper limits for the Durbin-Watson statistics with 238 observations are 1.61 and 1.74; they are 1.738 and 1.799 in the extended tables of Savin and White [49], and the lower limit is 1.728 in the Farebrother [14] tabulations when there is no intercept in the regression as here. The corresponding values for the 0.01 significance level are (1.48, 1.60), (1.63, 1.704), and 1.634.

Table 1(b)

FIML estimates for model (26) of long-term bond rates of return[1/] obtained by using: (a) the MOMENTS program, and (b) the LISREL IV program, over the period March 1959 to December 1978, with the value-weighted NYSE index as the market return.[2/]

	$\hat{\beta}_{12}$ (Std. Error)				$\hat{\beta}_{13}$ (Std. Error)			
	No restrictions on Disturbance Cov Matrix in (26)		Restriction: $Cov(u_1^j, u_2)=0$, $j=1,\ldots,6$ applied in (26)		No restrictions on Disturbance Cov Matrix in (26)		Restriction: $Cov(u_1^j, u_2)=0$ $j=1,\ldots,6$ applied in (26)	
Bond Portfolio	MOMENTS	LISREL IV	MOMENTS	LISREL IV	MOMENTS	LISREL IV	MOMENTS	LISREL IV
1	0.029 (0.045)	0.019 (0.015)	0.024 (35.512)	-0.0003 (0.0006)	1.097 (0.161)	1.126 (0.051)	1.081 (1.278)	1.066 (0.088)
2	0.064 (0.135)	0.041 (0.031)	0.049 (0.064)	0.0006 (0.0009)	1.124 (0.488)	1.267 (0.105)	1.074 (0.230)	1.133 (0.131)
3	0.090 (0.212)	0.117 (0.091)	0.080 (0.124)	0.007 (0.002)	1.037 (0.763)	1.577 (0.313)	0.999 (0.447)	1.173 (0.359)
4	0.061 (0.338)	0.214 (0.188)	0.089 (0.156)	0.030 (0.007)	0.862 (1.217)	1.895 (0.645)	0.963 (0.564)	1.099 (0.098)
5	0.058 (0.452)	0.346 (0.031)	0.085 (13.22)	0.051 (0.013)	0.778 (1.630)	2.298 (1.062)	0.874 (4.796)	1.022 (0.016)
6	0.020 (0.632)	0.343 (0.364)	0.070 (17.76)	0.076 (0.019)	0.708 (2.279)	1.156 (1.245)	0.890 (6.397)	0.786 (0.026)

1/ As defined in Table 1(1a).

2/ The indicator variables used in estimation of (26) are those given in Table 1(a), Footnote 2.

Table 2(a)

Test statistics for the asset pricing hypotheses[1/] listed in Section 5 derived from the instrumental variable and FIML tests based on model (26) for bond rates of return[2/] and the value weighted NYSE market index over the period March 1959 to December 1978.

	Instrumental Variable		FIML-MOMENTS		FIML-LISREL IV	
Hypothesis	X^2 or F	p-value	X^2	p-value	X^2	p-value
1	See Table 5[3/]				18.72	0.005
2	See Table 5[3/]		211.74	<0.001	109.47	<0.001
3	19.76	<0.005	973.91	<0.001	24.25	<0.001
4	24.74	<0.005	1074.15	<0.001	115.00	<0.001
6	See Table 6[3/]				NA[4/]	

1/ The restrictions on the model (26) implied by the asset pricing hypotheses are those listed in Section 5 of the text.

2/ Returns are defined in Table 1(a).

3/ Results are presented equation-by-equation in Table 5.

4/ NA means not available for reasons outlined in Section 5 of the text.

Table 2(b)

Test statistics for the asset pricing hypotheses[1] listed in Section 5 derived from the instrumental variable and FIML tests based on model (26) for bond rates of return[2] and deviations about the expected NYSE rate of return conditional on the riskless T-bill rate as the market rate of return.

	Instrumental Variable		FIML-MOMENTS	
Bond Portfolio	X^2 or F	p-value	X^2	p-value
1	See Table 5[3]		0.000	1.000
2	See Table 5		0.000	1.000
3	14.31	0.025	341.44	<0.005
4	14.46	0.025	NA[4]	
5	See Table 6		NA[4]	

1/ The restrictions on the model (26) implied by the asset pricing hypotheses are those listed in Section 5 of the text.

2/ Returns are defined in Table 1(a).

3/ Results are presented equation-by-equation in Table 5.

4/ NA means not available for reasons outlined in Section 5 of the text.

Table 3(a)

Estimates of the coefficients of the Hausman two-step instrumental variable procedure for testing exogeneity of the value-weighted NYSE market rate of return, after orthogonalization with respect to the one-month T-bill rate, in explaining long-term bond returns[1/] over the period March 1959 to December 1980.

$$\text{MODEL:}\quad R^{j}_{ja} = \beta_{12}\widehat{OR}_{ma}(t) + \beta^{j}_{13}R_{Ft} + \phi^{j}_{12}OV(t) + u_{j}(t) \qquad (27)$$

$$\widehat{OR}_{ma}(t) = \hat{\underline{q}}'\underline{Z}(t)^{2/}$$

$$OV(t) = R_{ma}(t) - \widehat{OR}_{ma}(t)$$

Bond Portfolio	$\hat{\beta}_{13}$ (Std. Error)	$\hat{\beta}_{12}$ (Std. Error)	$\hat{\phi}_{12}$ (Std. Error)	DW[3/]	R^2
1	1.103 (0.016)	0.023 (0.011)	0.002 (0.001)	1.827	0.95
2	1.121 (0.043)	0.065 (0.029)	0.015 (0.004)	1.800	0.75
3	1.074 (0.073)	0.112 (0.049)	0.030 (0.007)	1.804	0.50
4	1.045 (0.108)	0.123 (0.072)	0.045 (0.011)	1.881	0.33
5	0.960 (0.143)	0.079 (0.095)	0.060 (0.014)	1.887	0.21
6	0.963 (0.199)	0.049 (0.132)	0.079 (0.020	2.017	0.14

1/ $R^{j}_{a}(t) \equiv R^{j}(t) - [R^{j}(t) - E(R^{j}(t)|R_F, \pi(t-1), \ldots)][\pi(t) - E(\pi(t)|R_F, \pi(t-1), \ldots)]$ where $R^{j}(t)$ is the rate of return on a portfolio of bonds with maturity range j, $\pi(t)$ is the rate of inflation (consumer price index), and R_F is the (known) nominal rate of return on a T-bill with one month to maturity.

2/ The indicator variables: seasonally adjusted retail sales of nondurable consumer goods, industrial production of nondurable consumer goods, the average realized real rate of return on one-month T-bills over the past twelve months, the growth rate of the monetary base over the past twelve months, and an interpolated monthly series of quarterly per capital

2/ (Continued)
consumption, in addition to the exogeneous nominal one month T-bill return comprise the vector $\underline{Z}$ in these regressions.

3/ With three regressors, the 0.05 significance level lower and upper limits for the Durbin-Watson statistics with 238 observations are 1.61 and 1.74; they are 1.738 and 1.799 in the extended tables of Savin and White [49], and the lower limit is 1.728 in the Farebrother [14] tabulations when there is no intercept in the regression as here. The corresponding values for the 0.01 significance level are (1.48, 1.60), (1.63, 1.704), and 1.634.

Table 3(b)

FIML estimates for model (26) of bond rates of return[1/] obtained by using: the MOMENTS program over the period March 1959 to December 1978, with the deviations about the expected NYSE return conditional on the riskless rate as the market return.[2/]

Bond Portfolio	$\hat{\beta}_{12}$ (Std. Error) No restrictions on Disturbance Cov Matrix in (26) MOMENTS	$\hat{\beta}_{12}$ (Std. Error) Restriction: $Cov(u_1^j, u_2)=0$, $j=1,\ldots,6$ applied in (26) MOMENTS	$\hat{\beta}_{13}$ (Std. Error) No restrictions on Disturbance Cov Matrix in (26) MOMENTS	$\hat{\beta}_{13}$ (Std. Error) Restriction: $Cov(u_1^j, u_2)=0$ $j=1,\ldots,6$ applied in (26) MOMENTS
1	0.0445 (3.674)	0.0243 (0.013)	1.154 (13.226)	1.081 (0.049)
2	0.0528 (7.750)	0.0499 (0.018)	1.085 (2.799)	1.075 (0.065)
3	0.0844 (14.370)	0.0800 (0.033)	1.016 (5.172)	0.9996 (0.122)
4	0.0940 (22.638)	0.0889 (0.052)	0.981 (8.492)	0.9625 (0.195)
5	0.0215 (34.612)	0.0845 (0.086)	0.648 (12.459)	0.8745 (0.316)
6	-0.0162 (45.135)	0.0700 (0.113)	0.579 (16.247)	0.8899 (0.419)

1/ As defined in Table 1(1a).

2/ The indicator variables used in estimation of (26) are those given in Table 1(a), Footnote 2.

Table 4

Tests of the hypothesis $H_0(1)$ and $H_0(2)$ that the (assumed constant) coefficients of the instrumental variable and ordinary least squares regressions for bond rates of return in (26) are in accord with the restrictions imposed by the asset pricing models.

Bond Portfolio	F Statistic[1/] Test of $H_0(1)$	Test of $H_0(2)$
1	18.651	18.118
2	3.661	3.487
3	0.062	0.051
4	0.093	0.041
5	0.075	0.050
6	0.053	0.049

(a) Market Return Variable

	F Statistic[1/] Test of $H_0(1)$	Test of $H_0(2)$
1	18.224	17.89
2	3.400	3.385
3	0.051	0.047
4	0.042	0.038
5	0.092	0.044
6	0.047	0.049

(b) Orthogonalized Market Return Variable

1/ $F_{1,236}(95\%) \simeq 3.84$

2/ $F_{1,236}(99\%) \simeq 6.63$

Table 5

F-test statistics for the hypothesis that the real "betas" and conditional expected returns on long term bonds are constant over the period March 1959 to December 1978.

	Value Weighted Market Index		Orthogonalized Market Index	
Bond Portfolio	Instrumental Variable	OLS	Instrumental Variable	OLS
1	3.278	0.000	0.561	0.000
2	2.5155	0.967	0.372	0.870
3	0.1205	0.433	0.076	0.405
4	0.0836	0.348	0.060	0.312
5	0.0702	0.106	0.033	0.079
6	0.1332	0.589	0.354	0.512

1/ $F_{1,236}(95\%) \simeq 3.84$

2/ $F_{1,236}(99\%) \simeq 6.63.$

FOOTNOTES

1. The term "instrumental variable" is not used in this section strictly in its usual econometric meaning: here, it is a (possibly unobservable) index of optimal portfolio composition in the constant beta case.

2. Strictly, $E(R_j)$ should be written $E(R_j|R_F)$ in (1). For simplicity, the conditioning is made explicit only where necessary to avoid confusion.

3. Some writers include wealth as an opportunity set variable. This makes most sense when the economy's production function displays nonconstant returns to scale. In this paper, however, opportunity set changes will mean shifts in attainable production conditional on invested wealth or capital. If the production function is not linear homogeneous, a parameter encompassing changes in productivity attrributable to changes in the quantity of output (i.e., movements along the function) is included along with the state variables.

4. "Once a firm paradigm through which to view nature has been found, there is no such thing as research in the absence of any paradigm. To reject one paradigm without simultaneously substituting another is to reject science itself....there is no such thing as research without counter-instances." (Kuhn [30, p. 79]).

5. If $R_F(t)$ is endogenous, (10a) will still be identified by the covariance restriction involving $u_1(t)$ which will be necessary to make $R_F(t)$ a predetermined endogenous variable in (10a).

6. If meaningful aggregation is not possible, uncertainty would be inherent in the distribution of wealth changes and price level changes, but this is beyond the scope of this paper. In fact, the single price level index used here really only makes sense for homothetic preferences: "the fundamental and well-known theorem for the existence of a price index that is invariant under change in level of living....is that each dollar of income be spent in the same way by rich and poor, with income elasticities exactly unity (the homothetic case)." (Samuelson and Swamy [44, p. 568].

7. An assumption that the price level is nonstochastic (and known to the investor) would reduce the two sources of uncertainty to one, for in that case the price index is used only as a fixed translator of nominal magnitudes the (stochastic) real wealth variable.

8. In the sociology/psychometric literature, (26) is called a path model.

9. Robinson [39] formulates the MIMIC model more generally, so that (25) might become:

$$
\begin{aligned}
X_1(t) &= \gamma_1 \xi(t) + \underline{\alpha}_1' \underline{Z}(t) \\
&\vdots \qquad\quad \vdots \qquad\quad \vdots \\
X_k(t) &= \gamma_k \xi(t) + \underline{\alpha}'_k \underline{Z}(t) \\
\xi(t) &= \underline{\theta}' \underline{Z}(t) + u(t)
\end{aligned}
$$

where $\underline{Z}(t)$ is a vector of causes of $\xi(t)$ and $\underline{X}(t)$. In this extended model, the interdependence between indicators and causes is even more complex, as are any propositions needed to justify labels on X_i as indicators or causes.

10. MOMENTS was written by Bronwyn H. Hall [21] and uses the Berndt, Hall, Hall, and Hausman [4] scoring procedure.

11. LISREL IV uses a modified Fletcher-Powell algorithm (see Gruvaeus and Joreskog [20]). It had been slightly altered for use here in accordance with a proposal by Bard [2] that, for Davidson-like algorithms, the second derivative matrix be re-initialized if it becomes nearly singular.

12. It can be shown (Holly [25]) that, against "local alternatives" (i.e., those for which the true covariance σ_{12} converges to the null 0 at the rate $/T$ as the sample size T increases), Hausman's test has the same asymptotic power as the conventional likelihood, LM and Wald procedures for testing whether $\sigma_{12} = 0$. Note that since there are more nuisance parameters than the parameter of interest here (σ_{12}), his result requires only correlation between σ_{12} and those nuisance parameters.

13. The portfolios are similar in concept to those constructed and described by Bildersee [6,7]. Since bonds are "rolled across" portfolios or they approach maturity, the returns are not adjusted for taxes, and flower bonds are excluded. The average maturity of the bond portfolios and the total number of bonds in each are reasonably stable through time (more specific details are available upon request).

14. The likelihood ratio test, which will be a uniformly most powerful test if one exists, is asymptotically equivalent to the Lagrange Multiplier and Wald tests. The "Wald test" of coefficient equality in the Hausman exogeneity test uses the unrestricted regression coefficient estimates rather than the restricted and unrestricted likelihood estimates.

15. The MOMENTS algorithm could not improve upon the starting values after 68 iterations.

16. Statements like these are offered for the reader's guidance only, along with a caveat. Even with a knowledge of sample size, it is difficult to interpret them in the absence of a loss function and without some consideration of prior knowledge.

17. Some tests were repeated with two restrictions on the bond equation terms of the disturbance covariance matrix to reduce the number of free parameters. In the first, adjacent elements on all diagonals in the first six equations except the main were constrained to be equal, i.e., if σ_{ij} denotes the covariance between the disturbances of security equations i and j, then $\sigma_{ij} = \sigma_{i+k,j+k}$, $0 \leq k \leq \min[N-i,N-j]$. (That block of the matrix would look like a Toeplitz matrix if all the elements on the main diagonal were equal.) In the second, only the disturbances of the adjacent maturity bond portfolios were allowed to be non-zero. In both cases, LISREL IV based tests rejected all five hypotheses, but the restrictions themselves were rejected.

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